

**THE COST OF CAPITAL
TO A
PUBLIC UTILITY**

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and the equity of the existing shareholders is not changed. However, if $P > E$, part of the funds raised accrues to the existing shareholders. Specifically, it can be shown that

$$v = 1 - \frac{E}{P} \quad (2.8.9)$$

is the fraction of the funds raised by the sale of stock that increases the book value of the existing shareholders' common equity. Also, v is the fraction of earnings and dividends generated by the new funds that accrues to the existing shareholders.

A more rigorous derivation of v follows. If the market for a firm's new shares is perfectly competitive, the number of shares given to new shareholders during $t = n$ in return for Q_n dollars must satisfy two conditions. The first is that the new issue must be sold at the prevailing price per share at the time of the issue. The other condition is that the dividend expectation a new shareholder obtains should have a present value equal to Q_n , the money he invests, when discounted at the rate k . With r the return the utility earns on common equity investment, b the retention rate, and $(1 - v)Q_n$ the book value of the common equity obtained by the new shareholders, their dividend in $n + 1$ will be

$$D_{n+1}^* = (1 - b)r(1 - v)Q_n. \quad (2.8.10)$$

Once in the corporation the new shares are identical with the old shares. Their dividends also are expected to grow at the rate $br + vs$. Hence, the above two conditions are satisfied if

$$\begin{aligned} Q_n &= \sum_{t=n+1}^{\infty} \frac{(1 - b)r(1 - v)Q_n(1 + br + vs)^{t-n-1}}{(1 + k)^{t-n}} \\ &= \frac{(1 - b)r(1 - v)Q_n}{k - br - vs}. \end{aligned} \quad (2.8.11)$$

Dividing both sides of Eq. (2.8.11) by Q_n and solving for v , we obtain

$$v = \frac{r - k}{r - rb - s}. \quad (2.8.12)$$

It can be shown that Eqs. (2.8.12) and (2.8.9) produce identical values of v . The interesting property of Eq. (2.8.12) is that it makes clear that the cost of new equity capital is p for continuous new equity financing as well as one-shot new equity financing. When $r = k$, $v = 0$, and new stock financing at the rate s has no impact on P . Of course, if $r = k$ then $x = p$. When $r > k$, v is positive, and share price increases with s .

The assumption that a utility is expected to stock finance at the rate s has implications for the measurement of k . The yield at which a share with continuous growth at the rate g sells is

$$k = \frac{D}{P} + g, \quad (2.8.13)$$

the current dividend yield plus the expected rate of growth in the dividend. However, now $g = br + vs$ and not simply br . It also should be noted that continuous stock financing at the rate s poses problems similar to continuous retention at the rate b . When $k < br + vs$, the model breaks down in explosive growth. The above discussion of the resolution of the dilemma posed by $p < bx$ applies here. It also may have been noted from Eq. (2.8.12) that v is negative with $r > k$ when $r < rb + s$ or $r(1 - b) < s$. This is reasonable, although it may appear strange. Notice that $r(1 - b)$ and s are the outflow and inflow of funds due to dividends and stock financing expressed as fractions of the common equity. When $r(1 - b) < s$ the company is expected, in effect, to draw funds from stockholders for all future time. Clearly it is nonoptimal for a company to set $s > r(1 - b)$, and the case may be ignored.

2.9 Finite Horizon Model

We have seen that if $x > p$ and b and/or s are large we can have $k \leq g$, and our continuous growth models break down. A resolution of this dilemma consistent with the perfectly competitive capital markets assumptions is provided by withdrawing the assumption that the dividend is expected to grow at the current rate g for all future time. Specifically, a utility with a very large x reasonably will invest at a very high rate. The resultant high values